Electron-beam acceleration by cyclotron-autoresonance interaction

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Using the cyclotron autoresonance, electron beams gyrating along a static magnetic guide field can be accelerated to high energies by interacting with a high-intensity laser field. The energy exchange of the beam-laser system satisfies overall energy conservation.

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I. INTRODUCTION

Current concepts on charge-particle-beam acceleration often rely on the high field gradient of laser as the energy source. The configurations for acceleration can, in general, be divided into two categories as those that use a background dispersive medium, such as a plasma, and those that operate in free space. The second one has the advantage that parameter controls of the dispersive medium in the presence of the laser field can be neglected. One of the promising schemes of the latter category is to use a low-intensity transverse static magnetic [1–3] or electric [4–7] field that alternates in space with increasing irregular period to accelerate longitudinally propagating electrons along the laser beam. The interaction is based on the linear force in the single-particle equation of motion of the electrons. As expected, the energy gain or energy loss of the electron is sensitive to its initial phase with respect to the laser field. Although it is able to accelerate the beam electrons as a whole, this type of interaction leads to a large energy spread [1]. The feedback on the wave equation of the laser field is neglected. This is acceptable for very-low-density beams where pump depletion needs not to be considered. For high-density beams, energy saturation becomes an important issue.

Here we consider a cyclotron-autoresonance interaction scheme which is comprised of an electron beam undergoing helical trajectories along an externally imposed high-intensity axial static magnetic guide field. A circularly polarized electromagnetic wave with the same handedness propagating along the beam axis can interact with the beam resonantly through the cyclotron autoresonance effect. In the maser mode, a high-energy-density electron beam is used as the pump to generate coherent electromagnetic waves [8–11]. In the accelerator mode, the intense laser field is used to boost the beam electrons [12,13]. Unlike the static transverse field scheme, this interaction is a nonlinear parametric interaction among the laser field, the beam electrons, and the axial magnetic field that generates the helical beam trajectory. For this reason, a strong axial magnetic field is required. To reward the cost of a high field, the cyclotron-autoresonance interaction inherently yields a relatively cold high-energy beam.

II. CYCLOTRON-AUTORESONANCE INTERACTION

From the equation of motion, an electron beam of perpendicular and parallel momentum densities $\gamma mnv_\perp$ and γmnv_z with respect to the magnetic guide field $\mathbf{B}=B_0\hat{\mathbf{z}}$ and energy density γmnc^2, where γ is the Lorentz factor and n is the beam density, describes a constant amplitude time-independent helical trajectory given by

$$\mathbf{v}=v_\perp(-\sin\theta,+\cos\theta,0)+v_z\hat{\mathbf{z}} , \quad (1)$$

where $\theta=(\Omega/\gamma v_z)z=k_w z$ is the azimuthal angle of the trajectory on the xy plane and $\Omega=eB_0/mc$ is the cyclotron frequency. The trajectory defines a self-generated right circularly polarized wiggler with spatial period $\lambda_w=2\pi/k_w$. Let us consider a laser field with frequency ω_s and wave number k_s characterized by the vector potential $\mathbf{A}_s$ given by

$$\mathbf{A}_s=A(\sin(\psi_s+\phi),\cos(\psi_s+\phi),0) , \quad (2)$$

where $\psi_s=(k_s z-\omega_s t)$ is the harmonic phase and ϕ is the relative phase. Here A and ϕ are slowly varying functions of z and t. The corresponding electric and magnetic fields are

$$\mathbf{E}_s=(A\omega_s/c)(\cos(\psi_s+\phi),-\sin(\psi_s+\phi),0) , \quad (3)$$

$$\mathbf{B}_s=Ak_s(\sin(\psi_s+\phi),\cos(\psi_s+\phi),0) . \quad (4)$$

The resonant interaction between the electron beam and the laser field takes place when the Doppler shifted wave frequency sensed by the electron equals the spatial wiggler frequency

$$(\omega_s-k_s v_z)=k_w v_z=\Omega/\gamma . \quad (5)$$

By considering the relation

$$\gamma^2(1-\beta_\perp^2-\beta_z^2)=1 \quad (6)$$

and the wave dispersion relation $\omega_s=ck_s$, we have

$$\omega_s=[(1+\beta_z)/(1+\gamma^2\beta_\perp^2)]\gamma\Omega , \quad (7)$$

where Eq. (7) relates the wave frequency to the cyclotron frequency and the beam parameters. Regarding ω_s and Ω as fixed, the above condition leads to a quadratic equation in γ with

$$\gamma=\beta_\perp^{-2}\{(\Omega/\omega_s)+[(\Omega/\omega_s)^2-\beta_\perp^2]^{1/2}\} ,$$

which shows that the resonant condition can be satisfied for real γ only for beams with $\beta_\perp<(\Omega/\omega_s)$.

Under the resonant condition of Eq. (5), the radiation damping suffered by the electrons is of prime importance. From the energy equation

$$\frac{d}{dt}(\gamma mc^2)=-e\mathbf{v}\cdot\mathbf{E}_s ,$$

the evolution of the electron energy is governed by

$$\frac{d}{dt}\gamma=\beta_\perp\omega_s a\sin(\theta+\psi_s+\phi) , \tag{8}$$

where $a=eA/mc^2$ is the normalized vector potential and d/dt is the total time derivative. From the momentum equation

$$\frac{d}{dt}(\gamma m\mathbf{v})=-e\left[\mathbf{E}_s+\frac{1}{c}\mathbf{v}\times(\mathbf{B}+\mathbf{B}_s)\right] ,$$

the longitudinal component reads

$$\frac{d}{dt}(\gamma\beta_z)=\beta_\perp ck_s a\sin(\theta+\psi_s+\phi) . \tag{9}$$

Comparing Eqs. (8) and (9), we have

$$(E-cp_z)=\text{const} , \tag{10}$$

where $E=\gamma mc^2$ and $p_z=\gamma mv_z$. Rewriting the resonant condition in terms of E and p_z, it reads $(E-cp_z)=mc^2\Omega/\omega_s=$const, which is the same as Eq. (10). This shows that, once the resonant condition is met, the energy and the longitudinal momentum of the beam vary in such a way that the resonance is self-maintained. As the energy γ increases, the wiggler period λ_w increases accordingly so that the resonance is locked in. This is the main feature of the cyclotron autoresonance. We note that the interaction between the electron beam and the wave field is dictated by the ponderomotive phase $\psi=(\theta+\psi_s)$. Taking a total time derivative on ψ and recalling that $\tan\theta=\beta_y/\beta_x$, we have

$$\frac{d}{dt}\psi=-\Delta\omega+\frac{1}{\gamma\beta_\perp}(\omega_s-k_sv_z)a\sin(\psi+\phi) , \tag{11}$$

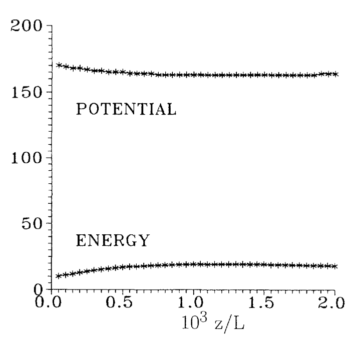

FIG. 1. The beam energy $\gamma(\xi)$ (lower curve) and the vector potential $A(\xi)$ (upper curve) are plotted against the interaction length ξ with $\beta_\perp=0.09$ and $(\omega_{pe}/\Omega)=0.1$.

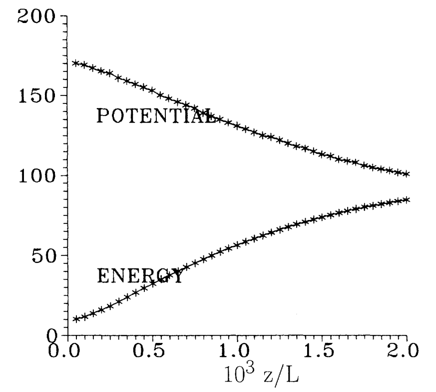

FIG. 2. The beam energy $\gamma(\xi)$ (lower curve) and the vector potential $A(\xi)$ (upper curve) are plotted against the interaction length ξ with $\beta_\perp=0.10$ and $(\omega_{pe}/\Omega)=0.1$.

where $\Delta\omega=[(\omega_s-k_sv_z)-\Omega/\gamma]$ is the frequency mismatch.

As for the wave field, we start from the wave equation

$$\left[\left[\frac{\partial}{\partial z}\right]^2-\left[\frac{1}{c}\frac{\partial}{\partial t}\right]^2\right]\mathbf{A}_s=-\frac{4\pi}{c}\mathbf{J} ,$$

where the term on the right-hand side represents the perpendicular electron current. Using the eikonal approximation, the slowly varying amplitude and relative phase are given by

$$\left[\frac{\partial}{\partial t}+c\frac{\partial}{\partial z}\right]a=-\tfrac{1}{2}(\omega_{pe}^2/\omega_s)\langle\beta_\perp\sin(\psi+\phi)\rangle , \tag{12}$$

$$a\left[\frac{\partial}{\partial t}+c\frac{\partial}{\partial z}\right]\phi=-\tfrac{1}{2}(\omega_{pe}^2/\omega_s)\langle\beta_\perp\cos(\psi+\phi)\rangle . \tag{13}$$

The interaction between the beam electrons and the wave field is through the ponderomotive phase ψ, which is periodic in space over a distance $\lambda_s=2\pi/k_s$. In steady state, it is therefore sufficient to follow the evolution of a

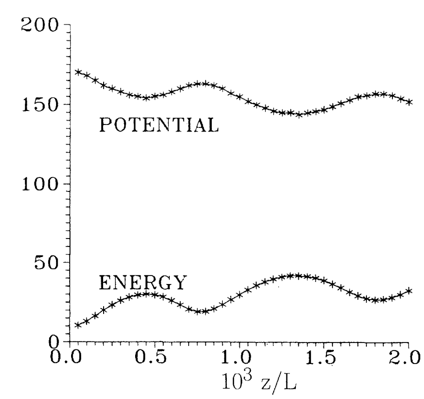

FIG. 3. The beam energy $\gamma(\xi)$ (lower curve) and the vector potential $A(\xi)$ (upper curve) are plotted against the interaction length ξ with $\beta_\perp=0.11$ and $(\omega_{pe}/\Omega)=0.1$.

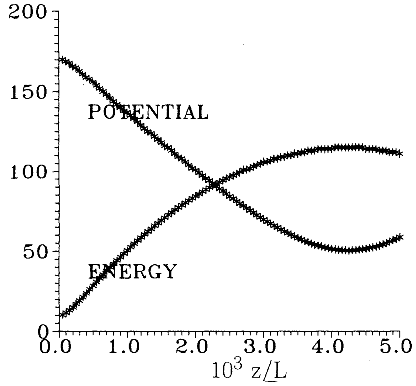

FIG. 4. The beam energy $\gamma(\xi)$ (lower curve) and the vector potential $A(\xi)$ (upper curve) are plotted against the interaction length ξ with $\beta_\perp=0.098$ and $(\omega_{pe}/\Omega)=0.1$

group of N sample electrons over such an interval λ_s. The initial phases ψ_i, with $i=1,2,3,\ldots,N$, can be assumed to be uniformly distributed over the interval $(-\pi,+\pi)$. The angular brackets on the right-hand side of Eqs. (12) and (13) stand for the phase averages over an ensemble of such N electrons. This allows a statistical description for the beam acceleration so that the results are not sensitive to the initial conditions.

Manipulating Eqs. (8) and (12) in steady state, we get the conservation of $[\omega_s^2 a^2+\omega_{pe}^2\beta_z\langle\gamma\rangle]$, which can be written as

$$\frac{1}{4\pi}E^2c+\langle\gamma\rangle mnc^2v_z=\text{const} . \qquad (14)$$

The apparent discrepancy of a factor of $\frac{1}{2}$ on the Poynting vector arises from the fact that we are dealing with circularly polarized wave of Eq. (3), where E is the amplitude of each component in the x and y directions. Therefore E is equivalent to the root mean squared amplitude of a corresponding linear wave. The conservation of the total energy flux consisting of the electron energy density flux $S_e=\gamma mnc^2v_z$ and the wave energy density flux $S_s=k_s^2A^2c/4\pi$ enables us to analyze pump depletion.

III. BEAM ACCELERATION

Let us define a reference length $L=c/\Omega$ and a reference frequency Ω to normalize distance z to L with $\xi=z/L$, frequency ω to Ω, and time t to Ω^{-1}. As an example for the cyclotron-autoresonant acceleration, we consider a laser intensity of $S_s=2.8\times10^{12}$ W/cm^2 with wavelength $\lambda_s=100$ μm. This gives a normalized vector potential $a=0.1$. Taking a guide field $B_0=100$ kG, the reference length and frequency are $L=1.6\times10^{-2}$ cm and $\Omega=1.76\times10^{12}$ rad/s. Considering an electron beam with density $n=10^{13}$ cm^{-3} such that $(\omega_{pe}/\Omega)=0.1$ and energy $\gamma=10$, the initial beam energy density flux is $S_e=2.4\times10^{11}$ W/cm^2. The beam energy $\gamma(\xi)$ and the vector potential $A(\xi)$ in units of cm G are plotted as a function of the interaction length ξ in Figs. 1–3. For $\beta_\perp=0.09$, Fig. 1 shows a weak coupling between the beam and the laser field. The beam energy rises from an initial $\gamma=10$ to $\gamma=20$ at $\xi=2000$. For $\beta_\perp=0.10$, Fig. 2 shows strong coupling between the beam and the laser field with linear energy growth for the beam throughout the whole interaction length reaching $\gamma=85$. For $\beta_\perp=0.11$, Fig. 3 shows that the interaction is already outside the linear growth region. The interaction is oscillatory with an overall average growth on the beam energy. In Fig. 4 we return to the linear growth case with $\beta_\perp=0.098$ and extend the interaction length to $\xi=5000$. In this case, the beam energy increases to a point that it surpasses the depleted wave energy and therefore the process is reversed almost at the end of the interaction region. Reducing the beam intensity by one order of magnitude so that $(\omega_{pe}/\Omega)=0.03$, the saturation is avoided, as in Fig. 5. Further reducing the density by another order of magnitude with $(\omega_{pe}/\Omega)=0.01$, similar results are shown in Fig. 6, where the laser pump remains constant over the interval.

To estimate the upper limit of the beam energy, we consider the cyclotron radiation loss. Using the following expression of the radiated power P:

$$P=(2e^2/3c)\gamma^6[(\dot{\boldsymbol{\beta}})^2-(\boldsymbol{\beta}\times\dot{\boldsymbol{\beta}})^2] \qquad (15)$$

and considering the electron velocity vector of Eq. (1), we obtain $P=(2e^2/3c)\gamma^2\beta_\perp^2\Omega^2$. From Eq. (8), the power in-

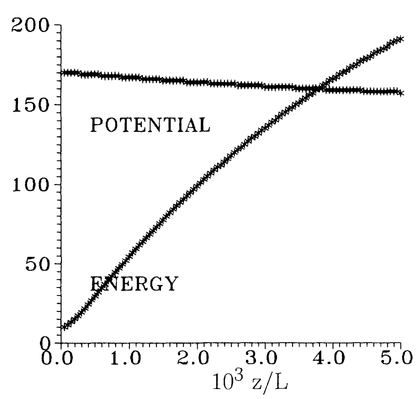

FIG. 5. The beam energy $\gamma(\xi)$ (lower curve) and the vector potential $A(\xi)$ (upper curve) are plotted against the interaction length ξ with $\beta_\perp=0.098$ and $(\omega_{pe}/\Omega)=0.03$.

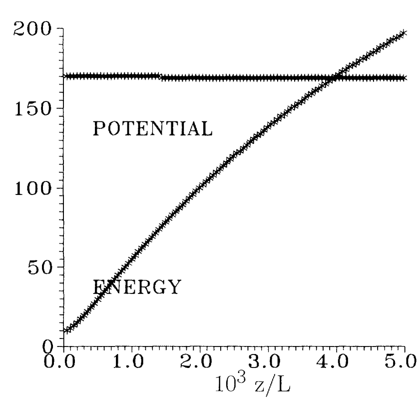

FIG. 6. The beam energy $\gamma(\xi)$ (lower curve) and the vector potential $A(\xi)$ (upper curve) are plotted against the interaction length ξ with $\beta_\perp=0.098$ and $(\omega_{pe}/\Omega)=0.01$.

put is estimated at $P = mc^2\beta_\perp\omega_s a$. Equating the two expressions leads to

$$\gamma^2 = (3/2\beta_\perp)(mc^2/e^2)(c/\Omega)(\omega_s/\Omega)a \ . \qquad (16)$$

With our parameters, the energy upper limit due to radiation loss is $\gamma = 10^6$, which is far superior to the limit imposed by Eq. (14) for energy density conservation. It is important to remark that, for $\xi = 2000$, the interaction length of this system is only $z = 32$ cm, which makes the system very compact.

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